Indian Statistical Institute, Bangalore

B. Math.(Hons.) I Year, Second Semester

Semestral Examination

Probability Theory-II (Back Paper)

Time: 3 hours Instructor: B.Rajeev

Maximum Marks 45

1. The joint density of X and Y is given by

$$f(x,y) = \begin{cases} e^{-(x+y)} & 0 < x < \infty, 0 < y < \infty \\ 0 & otherwise \end{cases}$$

Find the density function of the random variable $\frac{X}{Y}$. [8]

2. Suppose that the joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{e^{-x/y}\ e^{-y}}{y} & 0 < x < \infty, 0 < y < \infty \\ 0 & otherwise \end{cases}$$

Find $P\{X > 1 \mid Y = y\}$. [7]

3. Let X_1 and X_2 be independent standard normal random variables. Find the joint density of $X_1 + X_2$ and $X_1 - X_2$. [10]

4. An accident occurs at a point X that is uniformly distributed on a road of length L. At the time of the accident an ambulance is at a location Y that is also uniformly distributed on the road. Assuming that X and Y are independent, find the expected distance between the ambulance and the point of the accident. [7]

5. If n balls are randomly selected from an urn containing N balls of which m are white, find the expected number of white balls selected. [8]

6. Independent trials each resulting in a success with probability p are successively performed. Let N be the time of the first success. Find Var (N). [6]

7. Compute the moment generating function of a chi-squared random variable with n degrees of freedom. [6]

8. If 10 fair dice are rolled, find the approximate probability that the sum obtained is between 30 and 40, inclusive. [4]

9. Let $X_i, i = 1, \cdots, 10$, be independent random variables, each uniformly distributed over $(0, 1)$. Calculate an approximation to $P\left\{\sum_{i=1}^{10} X_i > 6\right\}$ Hint: you may use the approximation suggested by the central limit theorem. [4]